Multisymplectic Lie group variational integrators
Part 2: application to a geometrically exact beam in $\mathbb{R}^3$.

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ABSTRACT

The focus of this paper is to study and test a Lie group multisymplectic integrator (Part 1) for the particular case of a geometrically exact beam. We exploit the multisymplectic character of the integrator to analyze the energy and momentum map conservations associated to the temporal and spatial discrete evolutions. This allows us to explore the temporal motion of the beam and the spatial evolution of the wave motion through the beam. We discuss the necessary conditions to obtain a stable displacement in space versus time.

References

[1] Demoures F., Gay-Balmaz F., Kobilarov, M., and Ratiu T.S. [2014], Multisymplectic Lie algebra variational integrator for a geometrically exact beam in $\mathbb{R}^3$, to appear in *Commun. Nonlinear Sci. Numer. Simulat.*, dx.doi.org/10.1016/j.cnsns.2014.02.032

[2] Demoures F., Gay-Balmaz F., and Ratiu T.S. [2014], Multisymplectic variational integrators and space/time symplecticity, submitted.

[3] Kobilarov M. and Marsden, J.E. [2011] Discrete geometric optimal control on Lie groups, *IEE Transactions on Robotics*, **27**, 641–655.

[4] Lew A., Marsden, J.E., Ortiz, M., and West, M. [2003] Asynchronous variational integrators, *Arch. Rational Mech. Anal.*, **167**(2), 85–146.

[5] Marsden, J.E., Patrick, G.W., and Shkoller, S. [1998] Multisymplectic geometry, variational integrators, and nonlinear PDEs, *Comm. Math. Phys.*, **199**, 351–395.

[6] Marsden, J.E., and West, M. [2001] Discrete mechanics and variational integrators. *Acta Numerica*, Cambridge University Press, pp. 357-514.

[7] Simo, J.C. [1985] A finite strain beam formulation. The three-dimensional dynamic problem. Part I, *Comput. Meth. in Appl. Mech. Engng.*, **49**(1), 55–70.

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[8] Simo, J.C., Marsden, J.E., and Krishnaprasad, P.S. [1988] The Hamiltonian structure of nonlinear elasticity: the material and convective representations of solids, rods, and plates, *Arch. Rational Mech. Anal.*, **104**(2), 125–183.