Multisymplectic Lie group variational integrators
Part 1: derivation and properties.

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ABSTRACT

Multisymplectic variational integrators are structure preserving numerical schemes especially designed for PDEs derived from covariant spacetime Hamilton principles. The goal of this paper is to present a class of multisymplectic variational integrators for mechanical systems on Lie groups. The multisymplectic scheme is derived by applying a discrete version of the spacetime covariant Hamilton principle. The Lie group structure is used to rewrite the discrete variational principle in a trivialized formulation which allows us to make use of the vector space structure of the Lie algebra, via the introduction of a retraction map, such as the Cayley map. In presence of symmetries, we define the covariant momentum maps and derive a discrete version of the covariant Noether theorem. Some aspects of the symplectic character of the discrete temporal and spatial evolution will be given.

Further development and applications of this integrator to beam dynamics will be reported in Part 2.

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