Multisymplectic geometry: some perspectives

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Abstract

Since Fermat's principle in optics and Least Action principle in mechanics, the calculus of variation has been applied to almost all fundamental laws of mathematical physics with a great success. The resulting equations, known as the Euler-Lagrange equations, can be translated in the form of the Hamilton equations, the meaning of which is independent on the choice of coordinates and can be expounded geometrically. This means that one may sometime avoid messy computations. For variational problems with one variable (i.e. the time variable) this leads to the so-called symplectic geometry. For instance Noether's (first) theorem, which is one of the most important result of this theory, connecting symmetries and conserved quantities, has a particularly concise translation in symplectic geometry.
However the validity of the calculus of variation and of Noether's theorem is not limited to the calculus of variations with one variable and can be applied to problems with several variables, for instance the four coordinates of our space-time. An analogue of the symplectic geometry, called multisymplectic geometry, can be built. It leads to similar results but also to deep differences. We will present this setting and its motivations, in particular for understanding classical and quantum physics.

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